Estimating University Human Capital through Growth Models

Marisa Civardi, Emma Zavarrone[1]

Department of Methods of Economics, University of Milano - Bicocca, Italy

Summary. Our paper focuses on the law of growth of the human capital deriving from the evaluation of undergraduates' human capital due to university education. For this purpose, we introduce the definition of University Human Capital (UHC), a kind of human capital that sums up to the other kinds of human capital and that acts, for the concerned companies, as a detector of competences owned by graduates. It follows that UHC can be interpreted also as a component of the "intellectual capital" that characterizes the different kinds of enterprises. UHC individual growth trajectories are to be established by means of two-level growth models. We attempt to synthesise the law of individual UHC growth through both a logistic and a Gompertz function.
Keywords: Latent growth curves; Human capital; Specific competences; Multilevel models; Gompertz function.

1. Introduction

All the theories on Human Capital (HC) share a macroeconomics approach that raises questions about the suitability of the analysis of HC. Indeed, at a macroeconomics level, the focus of such an analysis is on assessing the general relevance in establishing the contribution to the national wealth. At a microeconomics level, instead, the focus is on quantifying the internal companies' HC. A microeconomics approach brings about the need to determine the capital identifying the company market value, the so-called *Intellectual Capital* (IC) (Lev, 2000).

[1] In this joint work, M. Civardi was responsible for the final editing of Sections 1, 2, 3 and 4, whereas E. Zavarrone was responsible for Sections 4.1, 5 and 6.

The rapidly changing job market requires graduates to hold a good post-graduate degree and the ability of being flexible for an effortless and a rapid vocational rehabilitation. University education and flexibility are not only two aspects of company IC, but also the value added by universities to their graduates. Therefore, identifying the HC that can be offered by university is crucial if an analysis and a quantification of the level of competence required by companies are being carried out.

In this perspective, a microeconomics approach to determine and measure UHC – that is the HC increase in terms of competencies for work by university – is a three-phase process. In the first phase, HC is assessed according to a university perspective. For each graduate the HC increase is quantified assessing the competence acquired by graduation.

In the second, HC is analysed according to a company's perspective, which implies that the required HC typologies are identified.

In the last phase, a feedback is dealt with: procedures and strategies developed by university to adapt higher education to market demand are analysed.

2 From Human Capital to Intellectual Capital

In order to adopt a microeconomics perspective to quantify UHC, an agreement on terminology is required to identify the common terms shared by universities and companies. To start with, while companies consider IC an intangible asset, universities regard it as the increase in graduates' knowledge by way of the learning process, which essentially transforms a secondary school graduate into a university graduate.

From the university viewpoint, UHC can be defined as the difference between students' HC when entering university and the HC held at graduation. In other words, UHC represents the improvement of knowledge, skills and attitudes attained thanks to the educational activities, use of didactic structures, exams passed and social interaction with fellow students. Measuring this improvement can be an aid to assess university ability to produce competences, depending on the variety of students' HC at university enrolment.

A first simplified representation of the higher education process can involve only two actors: users of education (students) and providers of education (university). A more realistic approach needs to take into consideration the existing relations between students' HC (family background, social background, and educational background), the education provider and companies in the job market. Obviously, the choices about education made by family, as well as all interactions with the social environment, crucially contribute to identify students' HC typology at the enrolment time (human capital at time t_0: HC_0).

During the time spent at the university, the initial HC_0 changes: it modifies and specializes according to the education received, the experiences, and the opportunities taken. It is exactly during higher education that HC specifies itself into University Human Capital. This kind of human capital assumes specific relevance and distribution in each student depending on his/her speed of learning and time spent at the university. As a result, at the end of the educational process, this component, which plays a differential role in the recruitment selection process, characterizes the graduates' human capital.

Moreover, IC can be considered as the HC component that characterizes company typologies. In fact, even though it is never included into companies' balance, IC is a company's intellectual asset and is at least as basic as capital assets. Undoubtedly, in today's economy, knowledge and information increasingly influence national wealth and consequently reduce the input of manufacturing.

Developing and managing knowledge in conjunction with intellectual capital is the most effective way through which companies can develop a competitive advantage: company's value and company's performance depend on its ability to create, gather and share knowledge. Companies, therefore, measure IC as the difference between its own market value and the cost of assets replacement.

Human capital, organizational capital and relational capital are the three main IC components. Organizational capital consists of company culture, company philosophy and information systems while relational capital consists of all relations with customers, suppliers and the environment/area/region. The main contributors to the development of human capital are, on the one hand, the knowledge, the capacity and the ability of each individual to provide clients with appropriate solutions, and on the other, the company overall ability to choose the most appropriate solution among all those proposed.

According to Edvisson & Malone (1997), *know-how* concerns two distinct aspects: knowledge and experience. With reference to graduate students, knowledge is anything formal and informal offered to the individual throughout university education. The cumulative experiences, regardless of their nature (deliberate or not, documentary, verbal, tacit), are the main sources of influence on learning and individual behaviour. In turn, individual performance is the result of the causal link between the two competences mentioned above through a circle linking motivations, characteristics, behaviour and result. The result influences in its turn motivation and characteristics.

3. UHC quantification

Assuming that some features of the above-mentioned HC characterize IC a whole, as a result, UHC can be considered as a very important element connecting university, which trains professional figures, and the job market, which searches them. In other words, UHC becomes the link between students, universities and companies.

Briefly, while students aim at gaining the credits required to graduate within the given time of their studies, university aims at providing effective educational services for training professional figures qualified to fill in the vacancies offered by companies. From this viewpoint, specifying HC in UHC represents the characterizing and decisive key element when companies select new employees or partners.

Let us now define the measure of UHC adopted in this work and its characteristics. UHC is the increase of students' competence acquired through higher education, in a dynamic context. This process consists of:

a. an *individual component* related to:
   1. different initial HC,
   2. different UHC increase paths,
   3. the interaction between individual UHC levels and the time spent to obtain them,
b. a *group component* that expresses the influence of the system on the individual component (such as the chosen study programme).

For a graduate *j*, the value of accumulated UHC at the time *t* is determined by:

$$UHC_j(t) = \frac{100\sum_{i=0}^{t} uhc_j(i)}{\max\ UHC_j(T_j)} \qquad t=1,2,3,\ldots,T, \qquad [1]$$

where: $uhc_j(i) = \sum cfu_j(i) \times mark_j(i)$ represents the global credits gained by *j* at the time *i* (the time when the examination result was registered by Student services) weighted with the respective marks. Hence, $uhc_j(i)$ represents the graduate *j* UHC increase at the observation *i*.

The denominator of [1] expresses the theoretical maximum for the subject *j* at the time T_j, that is, the time within which *j* obtains his degree. If no marks are added for first-class honours, the maximum for a three years degree graduate is: max $UHC_j(T_j)$ = 180×30. Therefore: $0 \leq UHC_j(T_j) \leq 100$.

4 Methodology

The individual component, which depends on individual students' UHC accumulation paths (component a.2), can be evaluated by assessing the parameters of appropriate time functions. Clearly, as this involves both a constant growth rate *k* and absence of an upper limit $\lim_{t \to \infty} f(t) = \infty$, the linear function $f(t) = \beta + kt$ is not the most appropriate to consider (Rao, 1958).

More realistically, by assuming an upper value α of UHC, several options are possible among which three are chosen. In the first one, the growth rate at time t is directly proportional to total growth to be still achieved, where the proportionality factor k>0. Under these conditions, the following growth curve is identified as a modified exponential growth curve:

$$f(t) = \alpha\left(1 - \beta e^{-kt}\right) \qquad \beta > 0 \quad . \qquad [2]$$

In [2], α is not only a function scaling factor, but it is the horizontal asymptote. Therefore, it is also UHC maximum value. In our case, $\alpha = 100$. Parameter k controls the scale along the time axis and, being UHC initial value UHC(0)=$\alpha(1-\beta)$, β determines the intersection between curve and vertical axis. If α is known, the transformed function $g(t)=f(t)-\alpha$ is an exponential function whose parameters can be estimated by the least squares method.

Provided the limit growth value equals α, the second option assumes that the growth rate at the time t is directly proportional to the total growth to be still achieved by the proportionality factor k>0. This leads to the following three-parameter *logistic function*:

$$f(t) = \frac{\alpha}{1 + \beta\, e^{-kt}} \qquad \beta > 0 \quad . \qquad [3]$$

This curve has two horizontal asymptotes: the lower is the time axis, the upper one is α, that is still UHC maximum value. The initial value is: $UHC(0)=\alpha/(1+\beta)$. In addition, the curve presents an inflection point.

As in [3] $f(t)$ reciprocal is a modified exponential function, introducing the function $g(t)=1/f(t)$ brings the identification of the logistic function back to identify the modified exponential function.

The third option, assumes that, given the upper limit α, time t growth rate is proportional to the difference between the maximum achievable level logarithm and the logarithm of the level achieved at the time t, with a proportional factor k>0. The corresponding function is *Gompertz curve*:

$$f(t) = \alpha\, exp\left(-\beta e^{-kt}\right) \qquad \beta > 0 \, . \qquad [4]$$

In Gompertz curve both the time axis and α are horizontal asymptotes. Therefore, α is again UHC maximum value, while the initial value is: $UHC(0)=\alpha\, e^{-\beta}$. In the Gompertz function, $log\, f(t)$ is expressed as a modified exponential function, hence, introducing the function $g(t)=log(f(t))$ brings the identification of its parameters back to identifying modified exponential function.

The three growth curves described above represent respectively three behavioural models for gaining the university credits required to obtain a degree. The assumption underlying the Italian Ministry of Education, University and Research supply of higher education is that students' yearly accumulation rate of credits in each of the three studying years, should be constant. In fact, this is not the case: few students obtain their degree within the expected time and when they do, a constant accumulation rate is very rare.

As already proved, the modified exponential function involves growth rates, and therefore number of credits gained in a unit time slot, decreasing in any time. According to this behavioural rule, students should gain more credits in their first year than in the following years. Unsurprisingly, its numerical application leads to the rejection of the model.

The other two models seem to be definitely more appropriate. In the second model, the hypothesis of proportionality between the relative growth rate and that to be achieved is equivalent to the assumption that the ratio between the credits gained in a time unit slot and the total credits at the beginning of the slot is proportional to the credits still to be gained out of the 180 required to obtain a degree. In the initial period of their studies, that is during the first examination sessions, students gain credits slowly while they speed up during the central periods.

According to Gompertz model, the ratio between credits gained during the generic time unit slot and the total credits at the beginning of the slot, that is the relative growth rate, is a decreasing linear function of the gained credits logarithm. Like the previous model, this also shows a slow start, but with an earlier acceleration. As we will shown below, both models fit the observed data.

4.1 Two-level growth linear models

In order to consider the effects of the interaction between UHC total amount (or its appropriate transformation) and the time spent achieving it, a 2-levels linear growth model can be referred to, where level 1 expresses the individual growth model. Because of what has been said above, three different growth models have been set up. They are based on the three individual UHC transformations introduced above:

UHC=log(100-UHC)* → growth model expressed by the modified exponential curve;

UHC=log(1/UHC-1/100)* → growth model expressed by the logistic function;

UHC=log [log100-logUHC]* → growth model expressed by the Gompertz curve.

If π indicates the model parameters for level 1 (“within” students) and β the parameters for level 2 (“amongst” students):

$$UHC_{ij}^{*} = \pi_{0j} + \pi_{1j} t_{ij} + r_{ij} \qquad r_{ij} \approx N(0, \sigma^2) \ , \qquad [5]$$

where

$$\pi_{0j} = \beta_{00} + u_{0j} \ ; \qquad \pi_{1j} = \beta_{10} + u_{1j} \ , \qquad [6]$$

and

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \approx N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right] \ . \qquad [7]$$

Equation [5] expresses the variable $UHC_{ij}*$ for each student j at time i as a linear function of time. Parameters π_{0j} e π_{1j} correspond respectively to the mean value at initial time and to $UHC_{ij}*$ growth rate. Equations [6] indicate that for student j $UHC_{ij}*$ value at initial moment (π_{0j}) depends on the average UHC_i* of all students' (β_{00}) and on a specific casual effect of student j (u_{0j}). Likewise, it indicates that the growth rate (π_{1j}) depends on the mean growth rate of all students (β_{10}) as well as a specific casual effect of student j (u_{1j}).

The presence of random errors on both levels shows the random nature of the model. In particular, as shown in [5], r_{ij} expresses instant time i casual effect on $UHC_{ij}*$ growth curve while, as said above, u_{0j} e u_{1j} are the random effects on student j growth curve in relation to all students' average one, respectively by intercept and growth rate.

Merging the two levels in one equation generates the following:

$$UHC_{ij} = \left[\beta_{00} + \beta_{10}t_{ij}\right] + \left[u_{0j} + u_{1j}t_{ij} + r_{ij}\right] \ . \qquad [8]$$

Equation [8] indicates that the multilevel model can be expressed as the sum of two parts. The first includes two fixed effects, one for intercept and the other for time parameter - growth rate. The second consists of three random effects: on intercept, on t slope, and on the residuals within students, r_{ij}. In this formula, both intercept and slope are assumed to be random. Additionally, at level 2, there is no variable. These assumptions can be both modified (Singer, 1998).

5. Application

The application was conducted on a cohort of University of Milan-Bicocca Faculty of Economics students, registered in the academic year 2000-2001. The career of each student was observed until December 31, 2004, provided everyone graduated within the observation period. Hence, the cohort was observed for 48 months.

The following characteristics were collected for each student:

- first examination mark (VE_1)
- credits associated with first examination (CE_1),
- first examination date (mm/yy) (DTE_1)
- second examination mark (VE_2)

- credits associated with second examination (CE_2)
- second examination date (mm/yy) (DTE_2)
- ....
- ....
- last examination mark (VE_n)
- credits associated with last examination (CE_n)
- last examination date (mm/yy) (DTE_n)
- final oral examination (graduation) mark (VL)
- credits associated with final oral examination (CL)
- final oral examination date (mm/yy) (DTL)

According to Student secretariat, the initial database contained, for each student, n_j+1 records (one record for the student' n_j examination, plus the final oral examination record). Each of n_j record consisted of: registration number, examination name, mark, credits and examination date; final oral examination record consisted of: registration number, graduation mark, credits and final oral examination date.

Consequently, the same monthly inquiry frequency and observation period was established for each student. The 31st of December 2000 was chosen as initial observation time t_0 because that is when freshmen can first sit for examinations and accumulate own UHC, while the maximum time T_{48} is December 31st, 2004.

A record of 49 inquiries was then built for each graduate, the first of which consists of the registration number (the identifier) while the other 48 include the $uhc_j(t_i)$ values at monthly inquiries $t_1, t_2, \dots, t_{48}$.

The $uhc_j(t_i)$ values were obtained from the following formula:

$$uhc_j(t_i) = \sum CE_j(t_i) \times VE_j(t_i) \ ,$$

where the sum operator has to do with examinations passed within the time interval between t_{i-1} e t_i . Hence, if student j passed two examinations in January 2001, his/her $uhc_j(t_1)$ is obtained from the summation of the two examination products (credits * mark); if he/she did not sit for any examination, his/her $uhc_j(t_1)$ is then equal to 0.

A new record based on 49 records and registration numbers was built including $UHC_j(t_i)$ values obtained as ratio between the cumulative of $uhc_j(t_i)$ and its theoretical maximum. The UHC growth curves for the five study programmes and for the whole faculty were calculated with these data.

The parameters of each of the three growth curves examined in Sections 4 and 4.1 were then evaluated with the procedure MIXED of SAS software (Littell *et al.*, 1996). With a focus of fixed effects (level 1), the first two columns on Table 1 show parameters of the functions (made linear in their parameters), on line one; and in brackets on line two, the related test t values. The other columns provide estimated UHC values at time 0 after 12, 24 and 36 months on line one, and on line two, the corresponding mean growth rates.

Table 1. Fixed effects for the three growth models: estimated averages.

Degree' Course	Linear Function parameters p_0	p_1	*UHC* estimated at time 0	*UHC* values (1° row) and Growth rate (2°row): after 12 months	after 24 months	after 36 months	Average *UHC* observed on 36 months
Exponential model							
1	4,968	-0,049	-43,728	20,585	56,120	75,755	69,758
t-value	*123,291*	*-9,682*		*3,926*	*2,169*	*1,199*	*14,800*
2	4,997	-0,056	-47,909	24,739	61,705	80,514	73,103
t-value	*161,576*	*-19,406*		*4,237*	*2,156*	*1,097*	*14,894*
3	5,016	-0,058	-50,830	24,689	62,397	81,224	77,960
t-value	*204,353*	*-17,846*		*4,359*	*2,176*	*1,087*	*12,142*
4	5,290	-0,084	-98,295	27,328	73,367	90,240	87,315
t-value	*66,989*	*-25,894*		*6,079*	*2,228*	*0,816*	*2,650*
5	4,993	-0,052	-47,375	21,246	57,916	77,511	73,434
t-value	*112,229*	*-11,055*		*4,113*	*2,198*	*1,174*	*14,740*
Faculty	**5,007**	**-0,056**	**-49,476**	**24,095**	**61,455**	**80,427**	**74,256**
t-value	***250,427***	***-27,981***		***4,286***	***2,177***	***1,105***	**14,277**
Logistic model							
1	-0,392	-0,149	1,459	8,090	34,359	75,686	69,758
t-value	*-1,239*	*-15,946*		*1,105*	*3,351*	*2,734*	*14,800*
2	-0,747	-0,130	2,066	9,156	32,504	69,705	73,103
t-value	*-3,385*	*-14,248*		*1,084*	*2,859*	*2,752*	*14,894*
3	-0,246	-0,155	1,263	7,635	34,811	77,527	77,960
t-value	*-0,609*	*-12,691*		*1,096*	*3,528*	*2,709*	*12,142*
4	-0,194	-0,162	1,200	7,848	37,397	80,733	87,315
t-value	*-0,172*	*-3,907*		*1,174*	*3,800*	*2,525*	*2,650*
5	0,010	-0,163	0,980	6,552	33,176	77,854	73,434
t-value	*0,030*	*-16,907*		*0,999*	*3,616*	*2,812*	*14,740*
Faculty	**-0,526**	**-0,141**	1,664	**8,435**	**33,397**	**73,185**	**74,256**
t-value	***-3,347***	***-22,928***		***1,091***	***3,141***	***2,771***	**14,277**
Gompertz							
1	1,583	-0,078	0,768	14,939	47,600	74,837	69,758
t-value	*23,331*	*-15,272*		2,226	2,769	1,700	*14,800*
2	1,793	-0,101	0,247	16,794	58,856	85,428	73,103
t-value	*19,012*	*-12,734*		*3,030*	*3,155*	*1,361*	*14,894*
3	1,772	-0,096	0,279	15,757	55,962	83,330	77,960
t-value	*17,262*	*-12,985*		*2,810*	*3,135*	*1,466*	*12,142*
4	2,388	-0,144	0,002	14,576	71,145	94,159	87,315
t-value	*9,424*	*-9,677*		*4,053*	*3,498*	*0,818*	*2,650*
5	1,668	-0,083	0,499	14,189	48,699	76,712	73,434
t-value	*23,072*	*-17,787*		*2,305*	*2,915*	*1,692*	*14,740*
Faculty	**1,775**	**-0,098**	0,273	**16,115**	**56,843**	**83,962**	**74,256**
t-value	***29,299***	***-19,229***		***2,875***	***3,139***	***1,435***	**14,277**

Since variables are absent from the model at level 2, the values above express mean values of UHC level and instant growth rate for single degrees and the Faculty. Test t values, which are all highly significant, particularly for the logistic and Gompertz model, allow us to reject the null hypothesis that parameters are equals to 0 in the population.

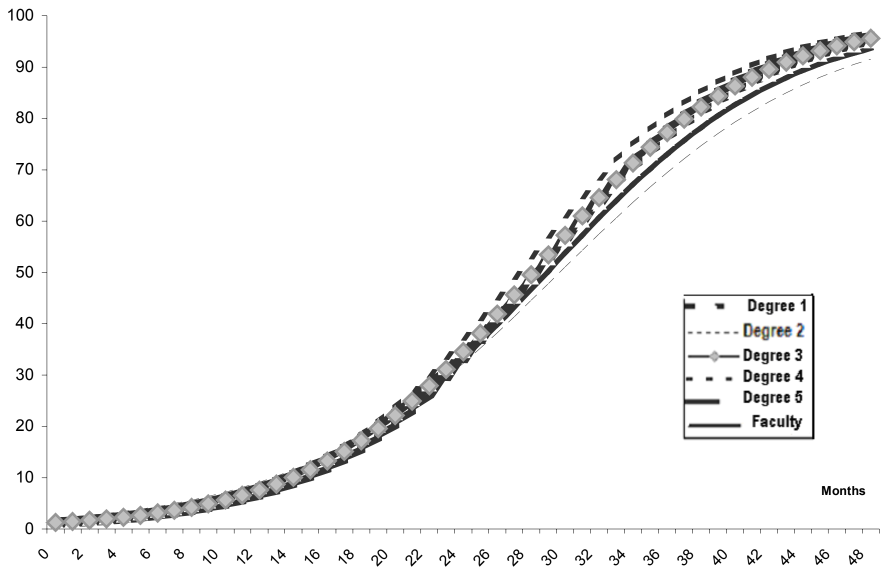

Figure 2 . Mean Trajectory for each Degree' Course and Faculty - Logistic model.

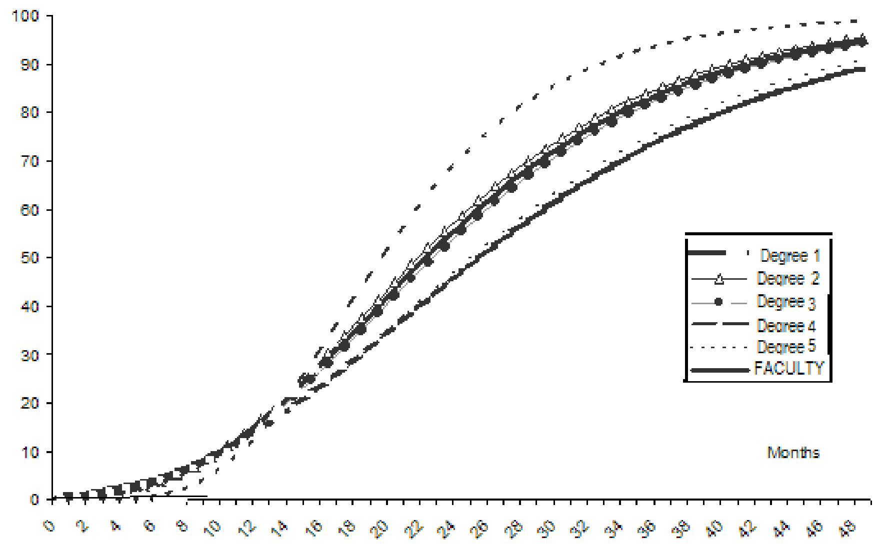

Figure 3. Mean Trajectory for each Degree' Course and Faculty for Gompertz model.

Figures 1 and 2 show the mean growth paths of each degree and the faculty, having assumed respectively a logistic model and a Gompertz model. Clearly, both models show slight differences in growth paths of study programmes. This may be due either to pre-selection, in other words, best student choose certain degrees and not others, or to a difference in the difficulty of each degree. The first hypothesis can be easily tested by introducing three individual characteristics (time independent variables x_{sij}, s =1,2,3) in equations [9], that is, type of secondary school diploma, secondary school final mark, university registration date.

Table 2 shows, instead, the evaluation of casual effects, particularly, of those related to intercept and slope and therefore to UHC mean levels and mean growth rate. The significance of these values shows the existence of intercept

Table 2. Comparison among fixed and random effects estimates

Degree' Course	Fixed Effects		Random Effects		
	Linear parameters		Covariance coefficients		
	β_{00}	β_{10}	τ_{00}	τ_{11}	τ_{01}
Logistic					
1	-0,392	-0,149	0.818	-0.018	0.001
t-value	*-1,239*	*-15,946*	*0.097*	*0.174*	*0.162*
2	-0,747	-0,130	2.298	-0.090	0.004
t-value	*-3,385*	*-14,248*	*0.000*	*0.000*	*0.000*
3	-0,246	-0,155	3.555	-0.099	0.003
t-value	*-0,609*	*-12,691*	*0.002*	*0.003*	*0.003*
4	-0,194	-0,162	3.080	-0.113	0.004
t-value	*-0,172*	*-3,907*	*0.418*	*0.412*	*0.410*
5	0,010	-0,163	1.556	-0.038	0.001
t-value	*0,030*	*-16,907*	*0.020*	*0.037*	*0.038*
Faculty	**-0,526**	**-0,141**	**2.003**	**-0.073**	**0.003**
t-value	*-3,347*	*-22,928*	*0.000*	*0.000*	*0.000*
Gompertz					
1	1,583	-0,078	0.043	-0.001	0.000
t-value	*23,331*	*-15,272*	*0.060*	*0.324*	*0.031*
2	1,793	-0,101	0.451	-0.036	0.003
t-value	*19,012*	*-12,734*	*0.000*	*0.000*	*0.000*
3	1,772	-0,096	0.224	-0.014	0.001
t-value	*17,262*	*-12,985*	*0.003*	*0.005*	*0.001*
4	2,388	-0,144	0.085	-0.007	0.001
t-value	*9,424*	*-9,677*	*no sig.*	*no sig.*	*no sig.*
5	1,668	-0,083	0.079	-0.003	0.000
t-value	*23,072*	*-17,787*	*0.012*	*0.112*	*0.006*
Faculty	**1,583**	**-0,078**	**0.317**	**-0.025**	**0.002**
t-value	*23,331*	*-15,272*	*0.000*	*0.000*	*0.000*

and slope variation that can be that explained with the individual characteristics, that is, with UHC level achieved by each student and through the time spent in achieving them. Covariance between intercept and slope, instead, is significant, although negative, only for two study programmes. These results indicate the existence of negative covariance between time and initial UHC so that longest times to achieve a given UHC level correspond to weaker students.

6. Conclusion

This work achieved the objective to obtain a measure of graduates specific competence growth during higher education based on results of each examination and related time dynamics of a cohort of students registered at five different degree courses of the University of Milan - Bicocca, Faculty of Economics who graduated within the 44 months of observation period. The objective was achieved with statistics methodologies recently presented in the area of variance components models.

The initial hypothesis is that the growth of competence corresponds to a particular type of human capital, namely, university human capital. This evolves in a dynamic context that obviously depends on students' characteristics (the individual component) and is affected by the type of degree course students' registered for (the group component).

On the one hand, the slightly different growth paths of different degree courses seem to confirm what the preliminary statistics analysis, concerned with type of degree and marks distribution, outlined: the existence of some sort of self-selection that brings clever students to choose certain courses rather than others. On the other hand, it seems to point at some disparity in the difficulty of different degrees.

In conclusion, it is this last hypothesis that needs to be further investigated if the effectiveness of education offered by a faculty is to be assessed.

References

EDVISSON L., MALONE M. (1997) *Intellectual Capital: Realizing your Company's True Value by Finding its Hidden Roots*, Harper Collins, New York.

LEV B. (2000) *Knowledge and Shareholder Value,* N.Y. University Press, New York.

LITTELL R., MILIKEN G., STROUP W., WOLFINGER R. (1996). *SAS Systems for Mixed Models,* North Carolina, SAS Institute.

RAO C.R. (1958) Some statistical methods for comparison of growth curves, *Biometrics*, **14**: 1-17.

SINGER J. (1998) Using SAS PROC MIXED to fit multilevel models, hierarchical models and individual growth models, *Journal of Educational and Behavioral Statistics*, **4**: 323-355.